Physics 11

Special Relativity Questions & Problems  (Answers)

1. If you were on a spaceship travelling at 0.50c away from a star, what speed would the starlight pass you?

   (The speed of light: 3.00 x 10⁸ m/s)

2. Does time dilation mean that time actually passes more slowly in moving references frames or that it only seems to pass more slowly?

   (It really passes more slowly; everything slows down relative to a non-moving reference frame.)

3. In the future a young astronaut may rush up to a an old grey-haired old man and calls out “Hi... how are you doing my son?. How might this be possible?

   (The astronaut went on a very high-speed spaceship and his time slowed down compared to his stay at home son.)

4. If you were travelling away from the Earth at 0.50c, would you notice a change in your heartbeat? Would your mass, height, or waistline change? What would observers on Earth using a telescope to see you say about you?

   (You would not notice anything different about yourself. To you everything seems normal. To a stationary observer your heartbeat, pulse and other life signs would all slowed down compared to those on earth. Your mass would have increase, and you would be skinner in the direction of motion.)

5. It is not correct to say that “matter can neither be created nor destroyed.” What must we say instead?

   (Energy cannot be created nor destroyed. Matter can be converted to energy and energy can be converted to matter.)

6. What happens to the relativistic factor when objects travel at normal everyday velocities?

   \[ \sqrt{1 - \frac{v^2}{c^2}} \]

   (The relativistic factor becomes equal to 1! This means that at ordinary speeds we do not to worry about it!)
7. A spaceship travels at 0.99c for 3 years ship time. How much time would pass on the Earth?

\[
t_o = t \times \sqrt{1 - \frac{v^2}{c^2}}
\]

\[
t_o = 3 \text{ y} \times \sqrt{1 - \frac{0.99^2}{c^2}}
\]

\[
t_o = 3 \text{ y} \times 0.141 = 0.423 \text{ y}
\]

8. A spaceship is travelling at 0.94c. It has been gone from the Earth a total of 10 years as measured by the people of the Earth. How much time passes on the spaceship during its travel?

\[
t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
t = \frac{10}{\sqrt{1 - \frac{0.94^2}{c^2}}}
\]

\[
t = \frac{10}{\sqrt{1 - 0.8836}}
\]

\[
t = 10 \text{ y} / 0.34117 = 29.3 \text{ y}
\]

9. A spaceship has been gone from the Earth for a total time of 5 years ship time. The people on the Earth have measured the time for the ship to be away to be 25 years. How fast was the ship travelling? (This is a challenge!)

\[
v = \sqrt{\frac{c^2 \times (t^2 - t_o^2)}{t^2}}
\]

\[
v = \sqrt{\frac{c^2 \times (25^2 - 5^2)}{25^2}}
\]
10. A meter stick is placed in a very high-speed spaceship. What length would the astronauts say the meter stick was? What would the length of the meter stick be as measured by stationary observers watching the spaceship and meter stick travel past them?

(Astronaut in high speed spaceship would say the meter stick is 1 m long. The stationary observers would say the meter stick is shorter than 1 m.)

11. A 520 m long (measured when the spaceship is stationary) spaceship passes by the Earth. What length would the people on Earth say the spaceship was as it passed the Earth at 0.87c?

\[ L = L_0 \times \sqrt{1 - \frac{v^2}{c^2}} \]

\[ L = 520 \text{ m} \times \sqrt{1 - \frac{0.87^2}{c^2}} \]

\[ L = 520 \text{ m} \times 0.493 = 256 \text{ m} \]

12. A 25 m long beam is shot past a stationary space station at 0.99c. What length does the people on board the space station measure the beam to be?

\[ L = L_0 \times \sqrt{1 - \frac{v^2}{c^2}} \]

\[ L = 25 \text{ m} \times \sqrt{1 - \frac{0.99^2}{c^2}} \]

\[ L = 25 \text{ m} \times 0.141 = 3.5 \text{ m} \]

13. A 100 m long steel beam is moving past the Earth. Observers on the Earth actually measure the steel beam to be only 50 m long. How fast was the beam travelling at? (This is a challenge!)

\[ v = \sqrt{\frac{c^2 \times (L_0^2 - L^2)}{L_0^2}} \]
\[ v = \sqrt{\frac{c^2 \times (100^2 - 50^2)}{100^2}} \]

\[ v = 0.87 \, c \]

14. A certain star is 10.6 light-years away (A light-year is the distance that light travels in one year. 1 Lt-\(y\) = 9.5 \(\times 10^{13}\) m!) How long would it take a spaceship travelling at 0.96c to reach the star:
   a. As measured by stationary observers on Earth?
      \[ t = \frac{d}{v} = \frac{10.6 \, \text{Lt-}y}{0.96 \, c} \]
      \[ t = 11.04 \, y \]
   b. As measured by observers on the spaceship?
      \[ t_o = t \times \sqrt{1 - \frac{v^2}{c^2}} \]
      \[ t_o = 10.6 \, y \times \sqrt{1 - \frac{0.96^2}{c^2}} \]
      \[ t_o = 10.6 \, y \times 0.28 = 3.0 \, y \]
   c. What is the distance travelled according to observers on the spaceship?
      \[ d = v \times t = 0.96 \, c \times 3.0 \, y = 2.88 \, \text{Lt-}y \]

15. A friend borrows your red “Ferrari Spaceship Model X-119” capable of travelling at 0.85c. The Ferrari is measured to be 5.6 m high and 18 m long in a stationary reference frame.
   a. How long would you say the Ferrari is as it sped by you at maximum speed?
      \[ L = L_o \times \sqrt{1 - \frac{v^2}{c^2}} \]
      \[ L = 18 \, m \times \sqrt{1 - \frac{0.85^2}{c^2}} \]
      \[ L = 18 \, m \times 0.527 = 9.5 \, m \]
b. Your friend has been gone for 2.0 h your time. How much time does he say has elapsed?

\[ t_o = t x \sqrt{1 - \frac{v^2}{c^2}} \]

\[ t_o = 2.0 \text{ h} \times \sqrt{1 - \frac{85^2}{c^2}} \]

\[ t_o = 2.0 \text{ h} \times 0.527 = 1.05 \text{ h} \]

16. What happens to the length of a spaceship if it could travel at the speed of light?

\( \text{(It would become infinitely short (it would have no length dimension)} \)

17. What happens to the mass of an electron as it is accelerated close the speed of light? Could the electron ever be made to travel at the speed of light? Explain why it can or can’t travel at 1.0c.

\( \text{It would gain an infinite amount of mass. No, it would take an infinite amount of energy to do so. The energy that you add to the electron to make it move at higher speeds becomes converted to mass!} \)

18. A 25 kg rock is accelerated to a speed of 0.98c.

a. What would the mass of this rock be at this speed?

\[ m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \]

\[ m = \frac{25 \text{ kg}}{\sqrt{1 - \frac{0.98^2}{c^2}}} \]

\[ m = \frac{25 \text{ kg}}{0.198} = 126 \text{ kg} \]
b. How much energy would be associated with the rock at rest? At this speed?

\[
\text{At rest: } E = mc^2 = 25 \times (3.00 \times 10^8 \text{ m/s})^2 = 2.25 \times 10^{18} \text{ J}
\]

\[
\text{At speed: } E = mc^2 = 126 \times (3.00 \times 10^8 \text{ m/s})^2 = 1.13 \times 10^{19} \text{ J}
\]

19. What speed would an object have to travel to increase its mass by 50%?
\[(v = 0.87 c)\]

20. It takes \(2.3 \times 10^{10}\) J of energy to operate a long train for 1.0 h. How long could you operate this train if 45 kg of matter could be converted to pure energy?

\[
(E = mc^2 = 45 \times (3.00 \times 10^8)^2 = 4.05 \times 10^{18} \text{ J}
\]

\[
\text{Time} = 4.05 \times 10^{18} \text{ J} / 2.3 \times 10^{10} \text{ J/h}
\]

\[
\text{Time} = 1.76 \times 10^8 \text{ h} = 20100 \text{ years!}
\]

21. What is the momentum of a 5.0 kg rock travelling at 0.99c? \((Be\ careful\ here!\)\)

\[
(p = mv, \quad \text{but} \quad m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

\[
m = \frac{5 \text{ kg}}{\sqrt{1 - \frac{0.99^2}{c^2}}}
\]

\[
m = \frac{5 \text{ kg}}{0.141} = 35.4 \text{ kg}
\]

\[
p = 35.4 \text{ kg} \times 0.99 \times 3.00 \times 10^8 \text{ m/s} = 1.05 \times 10^{10} \text{ kgm/s}
\]

22. A 12500 kg (rest mass) spaceship is travelling at 0.99c. What is the spaceship kinetic energy? \((Be\ careful\ here!\)\)

\[
(E_k = \frac{1}{2}mv^2 \quad \text{but} \quad m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}
\]
23. The Enterprise is travelling at 0.80c when it fires a missile at 0.95c relative to itself. How fast would the missile approach an asteroid as measured by a stationary probe?

\[
m = \frac{12500 \text{ kg}}{\sqrt{1 - \frac{0.99^2}{c^2}}}
\]

\[
m = \frac{12500 \text{ kg}}{0.141} = 88610 \text{ kg}
\]

\[
E_k = \frac{1}{2}mv^2 = 88610 \text{ kg} \times (0.99 \times 3.00 \times 10^8 \text{ m/s})^2
\]

\[
E_k = 3.90 \times 10^{21} \text{ J}
\]

\[
v' = \frac{v + u}{1 + \frac{v \times u}{c^2}}
\]

\[
v' = \frac{0.80 \text{ c} + 0.95 \text{ c}}{1 + \frac{0.80 \text{ c} \times 0.95 \text{ c}}{c^2}}
\]

\[
v' = \frac{1.75 \text{ c}}{1 + 0.76}
\]

\[
v' = 1.75 \text{ c}/1.76 = 0.994 \text{ c}
\]
24. An observer on Earth sees an alien spaceship approaching at 0.60 c. The Enterprise comes to the rescue overtaking the spaceship at 0.90c relative to the alien spaceship. How fast would the observers on earth measure the Enterprise to be travelling at?

\[
\begin{align*}
\mathbf{v'} &= \frac{\mathbf{v} + \mathbf{u}}{1 + \frac{\mathbf{v} \times \mathbf{u}}{c^2}} \\
\mathbf{v'} &= \frac{0.60 \text{ c} + 0.90 \text{ c}}{1 + \frac{0.60 \text{ c} \times 0.90 \text{ c}}{c^2}} \\
\mathbf{v'} &= \frac{1.5 \text{ c}}{1 + 0.54} \\
\mathbf{v'} &= \frac{1.50 \text{ c}}{1.54} = 0.974 \text{ c}
\end{align*}
\]